Bargaining over Incentive Contracts

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Abstract

Standard contract theory assumes that the principal holds all of the bargaining power. By implementing alternating offer and strategic delay, we transform the one-shot contracts design game into the infinite-horizon contracts bargaining game. The uninformed principal and the informed agent bargain over multiple dimensions. Our paper presents the following new results. When the difference between the agent’s types is sufficiently large, the efficient outcome is attained. However, when this difference is not sufficiently large, we attain either the "sequential separating equilibrium" or the "simultaneous separating equilibrium" depending on the parameter values. We prove the existence and uniqueness of the equilibrium. Lastly, we claim that the multi-dimensionality of bargaining helps to resolve the multiple equilibria problem of standard bargaining theory.

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1 Introduction

In many real-life situations, contracting involves bargaining. However, in standard incentive contract theory the principal is assumed to hold all of the bargaining power.\textsuperscript{1} She proposes and commits to contracts in the form of a take-it-or-leave-it offer and therefore requests a yes-or-no answer from the agent. The agent is not free to propose another one. If the agent rejects the offer, the interaction between them stops in the standard model, whereas in the real world it actually might continue.

The assumption that the principal holds all of the bargaining power is reasonable if there is only one principal and many agents, who compete for the principal’s offer. For instance, in the case of employment contracts, potential workers compete for one job offered by the firm. In such scenarios, it is reasonable to assume that the principal holds all of the bargaining power. However, in many real-life contracting situations, there is only one principal and one agent involved, and thus each party may have some bargaining power. For example, the firm and the labor union both have bargaining power when they negotiate the labor contract. In the bilateral monopoly situation, that is, one principal versus one agent, it may be reasonable to assume that the agent also enjoys some bargaining power, rather than that the principal holds all of the bargaining power.

In this paper, we relax the standard assumption by introducing bargaining into the incentive contract theory. More precisely, we implement alternating offers (Rubinstein, 1982) to the standard non-linear pricing model (Mussa and Rosen, 1978; Maskin and Riley, 1984). We assume that after rejecting the principal’s offer, the agent can propose new contracts. In turn, the principal can choose to accept or reject those new contracts. By introducing alternating offers, this research gives the principal and the

\textsuperscript{1}There are at least three excellent textbooks on contract theory, Salanie (1997), Laffont and Martimort (2002), Bolton and Dewatripont (2005). These books divide the theory into three categories: screening or adverse selection model, signaling model and moral hazard model. The first two are also called hidden information model, and the last is also called hidden action model. All three models are cast in the principal-agent framework.
agent approximately equal bargaining power. Now the principal and the agent are both essentially contract "designers". Thus, we transform the standard one-shot contracts design game into an infinite-horizon contracts bargaining game.\(^2\) In terms of bargaining, this is also multi-dimensional bargaining since the contracts usually specify more than one dimensions.

In addition to the alternating offers assumption, we also allow the players to strategically use the time between offers, following Admati and Perry (1987). In other words, each party, when it is his or her turn to move, can delay it as long as he or she wishes.\(^3\) We assume that until an offer has been made by the proposing player, the other player cannot revise previous offers.

In our contracts bargaining game, a seller (principal) and a buyer (agent) bargain over the sales contract which specifies the price and the quality of some goods. The seller has no private information, while the buyer does. In particular the buyer can have either a high or a low marginal valuation for the quality. The principal makes the first offer. Subsequently, the players make alternating offers with strategic delay until they reach an agreement. In rounds when the uninformed seller proposes the contracts, it is a screening problem. The seller can offer a menu of contracts, which consists of different combinations of price and quality, to induce the buyer to self-select. In rounds when the informed buyer proposes the contracts, it is a signaling problem. The buyer can use the combination of price, quality and strategic delay to signal his type. It is well known that games with incomplete information in which the informed player makes a move tend to have multiple equilibria. We implement the "intuitive criterion" (Cho and Kreps, 1987) to constrain out-of-equilibrium beliefs.

The following characterize the main results of this paper. If the difference between the two types of the buyer is sufficiently large, for all discount factors and all possible

\(^2\)Henceforth, we will call the "standard one-shot contracts design game" the "contracts design game", and call the "infinite-horizon contracts bargaining game" the "contracts bargaining game".

\(^3\)In the standard bargaining game, a fixed time between offers is specified exogenously. The assumption of strategic delay captures the idea that endogenous time between offers can be an important strategic variable in bargaining with incomplete information.
distributions of types, the complete information efficient outcome is achieved. The reason is that the sequential bargaining generates type-dependent continuation payoffs and the high-type will have a higher continuation payoff if he reveals his own type rather than if he mimics the low-type.

When this difference between the two buyer types is not sufficiently large, we attain either the “sequential separating equilibrium” or the “simultaneous separating equilibrium” depending on the initial proportion of types. If the initial probability of the low-type buyer is sufficiently small, then there exists a unique sequential separating equilibrium path. In this equilibrium the seller’s first offer is a single contract intended for the high-type, which is identical to the complete information contract. The high-type buyer accepts the seller’s first offer while the low-type buyer does not. The low-type buyer makes the counteroffer of his "least-cost-separating contract". In this counteroffer, the low-type can use the quality distortion or the strategic delay or combining both to signal his type. Hence, our newly defined "dynamic least-cost-separating contract" is more general than the usual static "Riley outcome"(Riley, 1979; Cho and Kreps, 1987), which only involves quality downward distortion in our case.

Furthermore, if the initial probability of the low-type buyer is sufficiently large, then there exists a unique simultaneous separating equilibrium path. In this equilibrium, the seller’s first offer is a menu of contracts from which the buyer can self-select. Both buyer types accept the menu without delay and each self-selects his preferred contract, thus revealing his type. Contracts characterized in this case are what we call "bargaining-proof contracts" with incomplete information.

After proving the existence and uniqueness of the equilibrium, we also extend our model to the finite many types case and the continuous types case.

This paper combines elements from the incentive contract literature and the bargaining literature. Before we proceed to the model, let’s briefly compare our contracts bargaining game with the standard contracts design game first and then with the standard bargaining game.

In the contracts design game, the contracts obtained should satisfy the participation
constraints (PCs)\(^4\) and the incentive constraints (ICs)\(^5\). In our contracts bargaining game, the contracts generated also meet these two constraints. Although the ICs of the contracts bargaining game are similar to those of the contracts design game, the PCs are not. For the contracts design game, the right-hand sides of PCs are outside options, which are usually normalized to zero. However, for the contracts bargaining game, the right-hand sides of PCs are the endogenous type-dependent continuation payoffs generated by the sequential bargaining. Finding the endogenous type-dependent continuation payoffs is the key to analyzing this type of bargaining game.

Standard bargaining theory focuses on one-dimensional bargaining, usually a buyer and a seller bargaining over the price of a good.\(^6\) In our contracts bargaining game, however, the buyer and the seller bargain over multiple dimensions, i.e., the price and the quality. If the quality is held constant in our model, then the price becomes the only dimension and our model degenerates to the standard one-dimensional bargaining. The main problem with the standard bargaining literature is that there are too many equilibria even after applying refinements. This main problem is one of the reasons that the incentive contract theory makes the "extreme" assumption that the principal has all of the bargaining power since the incentive contract literature wishes to focus on the incentive issue while avoiding the technical difficulty of multiple equilibria that arises from incomplete information bargaining.\(^7\)

\(^4\)Participation constraint is also called individual rationality constraint. It ensures that the agent wants to participate in the contract.

\(^5\)The incentive constraint is also called the incentive compatibility constraint or truth-telling constraint. It guarantees that the players tell the truth in the hidden information model or not to shirk or behave opportunistically in the hidden action model.


\(^7\)Myerson (1983) and Salanie (1997) cite this reason: "Because the issues of bargaining with incomplete information are so complicated, a good research strategy is to begin by just studying this case, where one individual has all of the bargaining ability." (Myerson, 1983, page 1767). "Unfortunately, the study of bargaining under asymmetric information is very complex, so much so that there is presently no consensus among theorists on what equilibrium concept should be used. The Principal-Agent model is a simplifying device that avoids these difficulties by allocating all bargaining
However, contrary to people’s thoughts, the multiple equilibria problem for the one-dimensional bargaining does not necessarily arise for multi-dimensional bargaining. If the multiple dimensions are correlated so that the players can combine those dimensions to either screen or signal for the types as we have done in this paper, the multiple equilibria problem of the standard bargaining literature can be mitigated or even resolved. We discuss this issue in more detail in section 5.1.

The basic structure of our model is developed in Section 2. Section 3 presents the efficient outcome with incomplete information. Section 4 characterizes the sequential and the simultaneous separating equilibrium and proves the existence of them with respect to different parameter values. Section 5 proves the uniqueness of the equilibrium described in section 4. Section 6 extends the two-type model to the cases with finite many and continuous types. Section 7 discusses possible extensions and then concludes.

2 The Model

We now present the model assuming the simplest environment in this section, but the results can be easily extended to a more general setup which we will discuss in section 6.

2.1 Objective functions and information

There are two parties, a seller (principal), denoted \( S \), and a buyer (agent), denoted \( \theta \), bargaining over the quality \( q \) and the price \( p \) of some goods. A feasible trade is denoted by \((q, p) \in [\mathbb{R}_+ \times \mathbb{R}_+]\). \( S \)'s production cost of quality \( q \) is \( C(q) \). Assume that \( C(\cdot) \) is twice continuously differentiable with \( C' > 0 \) and \( C'' > 0 \). \( C(0) = 0, C'(0) = 0 \) and \( \lim_{q \to \infty} C''(q) = \infty \).

However, Salanie (1997) also points out that bargaining is critical for the bilateral contracting and he said "since this is a bilateral monopoly situation, we cannot go very far unless we specify how the parties are going to bargain over the terms of exchange." (Salanie, 1997, page 5).
$S$ has no private information and her utility function is $V(q, p) = p - C(q)$. $\theta$’s utility function $U(q, p, \theta) = \theta q - p$, where the valuation for quality $\theta \in \{\underline{\theta}, \overline{\theta}\}$, is his private information. Assume $0 < \underline{\theta} < \overline{\theta}$. Therefore, we have $U(q, p, \overline{\theta}) > U(q, p, \underline{\theta})$ for all $(q, p)$, and $U(q, p, \overline{\theta}) - U(q', p, \overline{\theta}) > U(q, p, \underline{\theta}) - U(q', p, \underline{\theta})$ for any pair $q > q'$. This is the discrete form of the single-crossing property or the Spence-Mirrlees condition: at any given quality level, the high-type buyer is willing to pay more than the low-type buyer for the same increase in quality. Denote the proportion of $\underline{\theta}$ (the low-type) as $\pi$ and the proportion of $\overline{\theta}$ (the high-type) as $\bar{\pi}$, with $\pi + \bar{\pi} = 1$.

We define the first-best production level of $q$ to be that which maximizes the surplus under complete information about $\theta$. To do so, consider the following maximization problem:

$$\max_q R(q, \theta) = \theta q - C(q)$$

By the assumptions above, $q^*(\theta) = \arg \max R(q, \theta)$ exists and is uniquely determined by the first-order condition, $C'(q) = \theta$. $R^*(\theta) = R(q^*(\theta), \theta)$ is the maximum amount of surplus attainable when the seller bargains with a buyer of type $\theta$. Additionally, we have $\overline{R} > R^* = R^*(\theta) > 0$. We can see that even when the seller is dealing with the low valuation buyer $\underline{\theta}$, there are gains from trade.

### 2.2 The contracts bargaining game

The contracts bargaining game starts at time zero, at which time it is $S$’s turn to make an offer.\(^8\) Subsequently, players make alternating offers until they reach an agreement. $S$ can propose a menu of contracts $(\underline{m}, \overline{m})$ in the feasible finite set $W$,\(^9\) where $\underline{m} = (q, p)$, $\overline{m} = (\overline{q}, \overline{p})$,\(^10\) from which, after acceptance, $\theta$ is free to choose one contract from the

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\(^8\)The case in which $\theta$ makes the first offer is discussed in section 7.

\(^9\)The technical reason for requiring the contracts in $W$ to be finite is to ensure that a continuation equilibrium exists.

\(^10\)Since we allow only two types of buyer, without loss of generality, we can assume the menu offered consists of two contracts. The contracts in the menu could be identical, which would be the same as a single contract.
menu. After the rejection of the entire menu, \( \theta \) can only offer a single contract.\(^{11}\) If \( S \) rejects the contract, she again proposes a menu, and so on so forth. A response to an offer involves either an acceptance or a rejection, and it can be made right away or at any time later. Acceptance ends the game. But if a rejection is made, the game continues and the relevant player makes a counteroffer within no shorter than a given length of time, which is normalized to one. We assume that until a counteroffer is made, the other party cannot change its previous offer. The first offer in the game can be made at any time \( t \geq 0 \).

We use \( \tau \geq 0 \) to denote delay, i.e. the length of the time it takes the relevant player to either accept or reject an existing offer and make a counteroffer. A relevant history for the game is a sequence of unacceptable offers and a sequence of time lengths between offers. Formally, if \( N \in \{1, 2, \ldots, \infty\} \) is the number of rounds (offers), then a history of the \( N \) rounds is denoted by \( H^N = \{m^n, \tau^n\}_{n=1}^N \). Note that \( \tau^N \) denotes the time delay in round \( N \) since offer \( m^{N-1} \) was made; this history corresponds to the passage of \( t = \sum_{n=1}^N \tau^n \) time units.

A strategy for a player specifies for each history after which it is the player’s turn to move, the length of time delay \( \tau \), whether the latest offer is accepted and, if not accepted, a counteroffer. We consider only pure strategies.

An outcome of the game is \( M = (m(q, p), t) \), with the interpretation that, contract \( m \) is agreed on at time \( t \), the buyer pays \( p \) to the seller to obtain the goods with quality \( q \). The seller’s payoff is \( \delta^tV(m) \), while the buyer’s payoff is \( \delta^tU(m) \), where \( \delta \) is the discount factor.

The equilibrium concept we use is sequential equilibrium. In a sequential equilibrium, one specifies the strategies and also the beliefs of \( S \) for every history \( H^N \). We use \( \pi(H^N) \) to denote \( S \)’s belief at \( H^N \) that \( \theta = \theta \), and we use \( \pi \) to denote the entire system of beliefs. We require that updated beliefs be consistent and strategies be sequentially

\(^{11}\)Alternatively, one can assume that the buyer makes a menu offer. After the seller accepts the menu, the buy again selects his preferred contract from the menu. But this is not our bargaining rule here.
rational in the sense that, at every information set, a player’s strategy maximizes his or her expected payoffs, given his or her beliefs and the opponent’s strategy.

2.3 The standard non-linear pricing model

For reference, we first examine the standard non-linear pricing model. We characterize the optimal mechanism by identifying \((p, q)\) and \((\bar{p}, \bar{q})\), and following the convention, we assume that outside options for both buyer types are zero. The seller’s problem can be completely formulated as follows:

\[
V = \max_{\{(p, q); (\bar{p}, \bar{q})\}} \{\pi [p - C(q)] + \pi [\bar{p} - C(\bar{q})]\}
\]

subject to

\[
\begin{align*}
\theta q - p & \geq \theta \bar{q} - \bar{p} \quad \text{(IC)} \\
\bar{\theta} q - \bar{p} & \geq \bar{\theta} \bar{q} - \bar{p} \quad \text{(IC)} \\
\theta q - p & \geq 0 \quad \text{(PC)} \\
\bar{\theta} q - \bar{p} & \geq 0 \quad \text{(PC)}
\end{align*}
\]

Since the objective function is concave, and all the constraints are linear, this is a well-behaved problem. It can be easily shown that, among the four constraints, \((\text{IC})\) and \((\text{PC})\) are binding. Solving this problem, we have the following standard results:

\[
q^N < q^*, \quad \bar{q}^N = \bar{q}^*, \quad \text{where } q^* = \{q^*, \bar{q}^*\} \text{ is the first-best quality levels. That is, the low-type gets a sub-efficient quality level, while the high-type gets an efficient quality level.}
\]

\[
p^N = \theta q^N, \quad \bar{p}^N = \bar{\theta} \bar{q}^* - (\bar{\theta} - \theta)q^N, \text{ where } (\bar{\theta} - \theta)q^N \text{ is the information rent. That is, the low-type gets zero surplus, while the high-type gets a positive surplus, the information rent.}
\]

The payoff for the seller is given by

\[
V^N = \pi (\theta q^N - C(q^N)) + \pi \{(\bar{R}^N - (\bar{\theta} - \theta)q^N\}
\]

Note that first-order condition gives

\[
C'(q) = \theta - \frac{\pi}{\bar{\theta}}(\theta - \bar{\theta}). \text{ For } C'(q) \geq 0, \text{ we should have } \pi \geq 1 - \frac{\theta}{\bar{\theta}}. \text{ This condition ensures that } \theta \text{ can be included in the menu. Furthermore,}
\]


we should have \( \pi \geq \frac{(\theta - q)q^N}{\delta q^N - C(q^N)} \); this condition guarantees that while including \( \theta \) in the menu, the surplus \( S \) gets from \( \theta \) can at least compensate the information rent she gives to \( \overline{\theta} \). Otherwise, it is optimal for \( S \) to propose a single contract designed for \( \overline{\theta} \). In other words, to sustain the above standard results, \( \pi \) should be sufficiently large.

3 Efficient Outcomes

3.1 Complete information outcome

As a benchmark, first consider the complete information Rubinstein (1982) outcomes. We can obtain complete information Rubinstein contracts, that is, the complete information first-best contracts with alternating offers made by the seller and the buyer respectively:

\[
\begin{align*}
\overline{m}_S^* &= (q^*, C(q^*) + \frac{1}{1+\delta} R^*) \\
\underline{m}_S^* &= (q^*, C(q^*) + \frac{1}{1+\delta} R^*) \\
\overline{m}_B^* &= (\bar{q}^*, C(\bar{q}^*) + \frac{\delta}{1+\delta} R^*) \\
\underline{m}_B^* &= (\bar{q}^*, C(\bar{q}^*) + \frac{\delta}{1+\delta} R^*)
\end{align*}
\]

(1) and (2) are the complete information Rubinstein contracts that the seller provides to the high-type and the low-type buyer, respectively. (3) and (4) are the complete information Rubinstein contracts for the seller proposed by the high-type and the low-type buyer, respectively. Since this is a case of complete information bargaining, all of the quality levels in these contracts are first-best and the prices divide the total surplus between the seller and the buyer. Following Rubinstein (1982), we can get the following proposition:

**Proposition 1** When the seller has complete information about the buyer, there is a unique subgame perfect equilibrium. In this equilibrium, the seller offers \( \overline{m}_S^* \) (\( m_S^* \)) to \( \overline{\theta} \) (\( \theta \)) whenever it is her turn to make an offer, and accepts an offer \( \overline{m}_B^* \) (\( m_B^* \))
of buyer $\bar{\theta} (\theta)$ if and only if $V(m_B) \geq \frac{\delta}{1+\delta} R^* (V(m_B) \geq \frac{\delta}{1+\delta} R^*)$; buyer $\bar{\theta} (\theta)$ always proposes $m^*_B (m^*_B)$, and accepts an offer $m_S (m_S)$ if and only if $U(m_S) \geq \frac{\delta}{1+\delta} R^* (U(m_S) \geq \frac{\delta}{1+\delta} R^*)$. The outcome is that the seller proposes $m^*_S (m^*_S)$ at time zero, and the buyer $\bar{\theta} (\theta)$ immediately accepts this offer.

In the equilibrium of the complete information contracts bargaining game, the seller gets $\frac{1}{1+\delta}$ share of total surplus $R^* (R^*)$, and the buyer $\bar{\theta} (\theta)$ gets $\frac{\delta}{1+\delta}$ share of total surplus $R^* (R^*)$.

### 3.2 Efficient outcome with incomplete information

Now we return to the incomplete information scenario. It is simple to show that the low-type buyer will never mimic the high-type buyer’s complete information strategy since $U(m_S^*) > U(m_S)$ and $U(m_B^*) > U(m_B)$. Then the question is whether the high-type will mimic the low-type’s complete information strategy or not. The answer is classic: it depends.

Different from the standard contract design game, the alternating offer bargaining creates endogenous type-dependent continuation payoffs. The high-type can attain a higher continuation payoff by revealing his type than mimicking the low-type under certain conditions, that is, $U(m_S^*) \geq U(m_S^*)$ and $U(m_B^*) \geq U(m_B^*)$. The following proposition characterizes this condition:

**Proposition 2** For all $(\pi, \delta) \in (0, 1) \times (0, 1)$, there exists $c^0 \in (1, \infty)$ such that if $\frac{\pi}{\delta} \geq c^0$, the contracts bargaining game has a unique equilibrium outcome where the seller proposes $(m^*_S, m^*_S)$ at time zero, and the buyer $\bar{\theta} (\theta)$ immediately accepts this offer and selects $m^*_S (m^*_S)$.

That is to say, if the difference between the two types is sufficiently large, the high-type buyer essentially has a much higher continuation payoff by telling the truth than mimicking the low-type. Additionally, the low-type will never mimic the high-type,
hence the outcome is identical to the complete information Rubinstein outcome and the first-best quality level is achieved.\footnote{In different environments, Sen (2000) and Inderst (2003) get similar efficient results and Inderst (2003) has proven the uniqueness of the efficient result. Our proof of the uniqueness of the efficient result could be based on Inderst (2003) with some modification.} The following is a simple but formal proof.

**Proof.** There exists some $c$ that satisfies the following inequalities: $U(m^*_S) \geq U(m^*_S)$ and $U(m^*_B) \geq U(m^*_B)$. By letting $U(m^*_S) = U(m^*_S)$, we can find $c^0$. \hfill $\blacksquare$

This result sharply contrasts the result of the standard contract design game, in which the high-type mimics the low-type for sure.\footnote{The standard literature usually normalizes all types’ outside options to zero. If the outside options are type-dependent, many interesting cases could arise. (See Laffont and Martimort, 2002, page 101-105).} As in the standard non-linear pricing model which we discussed above, the contracts designed distort the low-type’s quality downward to prevent the high-type from mimicing the low-type and give the high-type information rent to induce him to tell the truth.

This outcome also contrasts the result of the standard bargaining literature, which typically ends up with multiple equilibria among which none are efficient.

This efficient outcome depends on the difference between the types. In the finite many type case, as long as the difference between any two adjacent types is sufficiently large, this result holds. However, in continuous type case, this result doesn’t hold.

### 4 Separating Equilibrium: Sequential vs. Simultaneous

From henceforth, let $\frac{\bar{\theta}}{\bar{\theta}} < c^0$, or $U(m^*_S) < U(m^*_S)$, that is, let the high-type have the incentive to mimic the low-type’s complete information strategy. Additionally, we make Assumption 1 in the spirit of the "intuitive criterion", and for the rest of the paper, we only discuss the sequential equilibrium that survives Assumption 1.

**Assumption 1 (A1).** Suppose that, according to the equilibrium strategies, either (i) at time $t$, $S$ accepts an offer $m$ made by $\theta$, or (ii) at time $t$, $\bar{\theta}$ accepts an offer $m$.
made by $S$. If $\theta$ offers $\tilde{m}$ at time $t + \tau$, and $(\tilde{m}, \tau)$ satisfies $\overline{U}(\tilde{m}, t + \tau) > \overline{U}(m, t)$, $\overline{U}(\tilde{m}, t + \tau) < \overline{U}(m, t)$ and $V(\tilde{m}, t + \tau) \geq V(m_B^*, t + \tau)$, then the belief of $S$ upon getting this offer must be $\pi = 1$.

In words, if an out-of-equilibrium strategy by the buyer is observed, no matter whatever beliefs the seller adopts after observing this deviation, her best response can only give the high-type (the low-type) a lower utility than what he gets in equilibrium, therefore, the seller believes the buyer is of high-type (low-type) with zero probability. Note that when the deviation $(\tilde{m}, \tau)$ happens, for the deviation to be meaningful, it also requires that $V(\tilde{m}, t + \tau) \geq V(m_B^*, t + \tau)$. In other words, the meaningful deviation should satisfy the seller’s participation constraint. Otherwise, no information is transmitted to the seller and the seller will ignore such kind of deviation.

We also assume $\delta \to 1$ from now on.

### 4.1 Least-cost-separating contracts

Since the high-type has the incentive to mimic the low-type and delay is costly, it might be in the low-type’s interest to separate himself from the high-type by the contract offer. The low-type has many ways to signal his type: distortion of $q$ downward, strategic delay or a combination of both. For this purpose, we first define the "least-cost-separating contracts" (Riley outcome). We focus on the Riley outcome because the "intuitive criterion" selects it in the two-type signaling model.

The usual static "least-cost-separating contracts" are obtained by successively solving the following programs:

Program for $\overline{\theta}$:

$$\max_{\{\overline{q}, \overline{p}\}} \overline{U} = \overline{\theta} \overline{q} - \overline{p}$$

$$\text{s.t. } \overline{p} - C(\overline{q}) \geq \frac{\delta}{1 + \delta} \overline{R}^*$$

(PC)
Program for θ:  
\[
\max_{\{q, p\}} \ U = \theta q - p 
\]  
\[ (5) \]

\[
\begin{align*}
\text{s.t. } & p - C(q) \geq \frac{δ}{1 + δ} R^* \\
& \bar{θ}q - \bar{p} \geq \bar{θ}q - p
\end{align*}
\]  
\[ \text{(PC)} \]
\[ \text{(IC)} \]

The right-hand sides of PCs are the continuation payoffs for the seller, which are identical to those of complete information. Once the low-type signals his type, the high-type and the low-type are separated, and then the seller has complete information so she should get the payoffs under complete information.

The solution to θ’s program yields \( \bar{m}_B^* \), which is the complete information Rubinstein contract. Substituting \( \bar{m}_B^* \) into θ’s program, we find \( m_B^* = (q^*, C(q^*) + \frac{δ}{1 + δ} R^*) \) with \( q^* < q^* \). In words, similar to the screening problem, the low-type distorts his own quality level downward to prevent the high-type from mimicking.

Note that null contract \((0, 0)\) is a possible solution for the low-type simply because the total surplus after signaling can be so small that the low-type even cannot afford to pay for the seller’s complete information payoff. Without other signaling device, the low-type simply will drop out.

Now we define the "dynamic least-cost-separating contracts" since θ can also separate himself by strategic delay in our model. The low-type buyer’s problem becomes:

\[
\max_{\{q, p\}, \tau} \ U = \delta^\tau \{θ q - p\} 
\]  
\[ (6) \]

\[
\begin{align*}
\text{s.t. } & p - C(q) \geq \frac{δ}{1 + δ} R^* \\
& \bar{θ}q - \bar{p} \geq \delta^\tau (\bar{θ}q - p)
\end{align*}
\]  
\[ \text{(PC)} \]
\[ \text{(IC)} \]

In (6), the low-type chooses \( \{(q, p), \tau\} \), which is one more dimension than (5). Solving the problem, we find \( M_B^* = \{m_B^*(q^*, C(q^*) + \frac{δ}{1 + δ} R^*), \tau^\*\} \). Depending on the
parameter values, this problem can have the interior solution, which is $q^* > 0$ and $\tau^* > 0$, or the corner solution, which is either $q^*_L > 0$ and $\tau^* = 0$ or $q^*_H = 0$ and $\tau^* > 0$. We have $U(m^*_B) \leq U(M^*_B)$. That is to say, the payoff for the low-type in the "static Riley outcome" $U(m^*_B)$ is also smaller or equal to that of the "dynamic Riley outcome" $U(M^*_B)$. These results are quite straightforward since program (6) should give the low-type at least as much payoff as program (5).

4.2 Sequential vs. simultaneous separating

Depending on the initial proportion of types, there are two possible separating equilibrium outcomes: the sequential separating equilibrium outcome (SEE) and the simultaneous separating equilibrium outcome (SIE). The following define the SEE and the SIE:

**Definition 1** Under the SEE, $S$ first provides $m^*_S$, and $\bar{\theta}$ accepts it without delay, but $\theta$ doesn't accept it. Instead $\theta$ counteroffers $m^*_B$ with strategic delay $\tau^*$. $S$ accepts the counteroffer without delay.

**Definition 2** Under the SIE, $S$ first provides a menu $(m^*_S, m^*_B)$, and both buyer types accept the menu without delay, then each type self-selects his own contract.

In the SEE, the seller first provides a single contract only to the high-type, while in the SIE, she provides a menu. If the initial probability of the low-type $\pi$ is sufficiently small, it is better for the seller to just exclude the low-type in the first offer rather than to include the low-type. If the seller includes the low-type in the initial offer, then the seller has to pay the high-type buyer information rent. However, if $\pi$ is sufficiently large, it is in the seller’s interest to provide a menu in order to include both types in her first offer.

In the SEE, there is a delay $\tau^*$ since the low-type implements $M^*_B$ to signal his type, while there is no delay in the SIE.

In the SEE, the seller’s expected payoff is $V^{SEE} = \pi \frac{1}{1+\delta} R^* + \pi \delta^{\tau^*} \frac{\delta}{1+\delta} R^*$. The first term is the expected payoff generated by trading with the high-type, and the second
term is the expected payoff generated by trading with the low-type. The seller gets her complete information continuation payoffs under both situations since types are separated sequentially.

To calculate the seller’s expected payoff for the SIE, we first derive the menu \((m^{s}_{S}, m^{S}_{S})\). The seller’s problem in the SIE is:

\[
V^{SIE} = \max \{ (p - C(q)) + \pi[p - C(\bar{q})] \}
\]

subject to:

\[
\begin{align*}
\theta q - p &\geq \theta \bar{q} - \bar{p} \\
\bar{q} - p &\geq \bar{q} - \bar{p}
\end{align*}
\]

(\text{IC})

\[
\begin{align*}
\theta q - p &\geq \delta U(M^{*}_{H}) \\
\bar{q} - \bar{p} &\geq \frac{\delta}{1 + \delta} R^{*}
\end{align*}
\]

(\text{PC})

For \(\text{PC}\), if the low-type rejects the seller’s offer, he will propose his dynamic least-cost-separating contract \(M^{*}_{B} = \{ m^{*}_{B}(q^{s}, C(q^{s}) + \delta \frac{\delta}{1 + \delta} R^{*}, \tau^{*}) \}\), and his payoff will be \(\delta U(M^{*}_{H})\). Since delay is costly for the seller, if she wishes the low-type to accept her offer, the contract designed for the low-type should give him at least his continuation payoff \(\delta U(M^{*}_{H})\). The right-hand side of \(\text{PC}\) is the high-type’s continuation payoff if the two types are separated.

We can show that \(\text{IC}\) and \(\text{PC}\) are binding. Solving the problem, we can get contracts \(m^{s}_{S}(q^{N}, \theta q^{N} - \delta U(M^{*}_{H})), m^{S}_{S}(\bar{q}^{N}, \bar{q}^{N} - (\bar{q} - \theta)q^{N} - \delta U(M^{*}_{H}))\), and the seller’s payoff \(V^{SIE} = \pi(\theta q^{N} - C(q^{N})) + \pi((\bar{R} - (\theta - \bar{\theta})q^{N}) - \delta U(M^{*}_{H})).\)

It is interesting to compare the SIE program above with the standard non-linear pricing model in section 2.3. The objective functions are exactly the same for both, as well as the ICs. The only difference between the two problems is the right-hand sides of the PCs. The right-hand sides of PCs for the standard non-linear model are the outside options, while for the SIE they are the endogenous type-dependent continuation payoffs for the low- and the high-type, respectively. The qualities specified in both menus are exactly the same for the high-type and the low-type, respectively, while the prices as
well as the income distribution are different. This is quite intuitive because in the SIE the agent has bargaining power, thus he should expect to get more.

Now by comparing the expected payoffs for the seller under the SEE and the SIE, we can find the cutoff value \( \hat{\pi} \).

**Proposition 3** There exists a \( \hat{\pi} \) such that if \( 0 < \pi \leq \hat{\pi} \), \( S \) is better off in the SEE than in the SIE; on the other hand, if \( \pi > \hat{\pi} \), \( S \) is better off in the SIE than in the SEE.

**Proof.** Let \( V^{SEE} \geq V^{SIE} \), we can find \( \pi \leq \hat{\pi} = \frac{\overline{\theta} q^N + \delta U(M^*_H) - \frac{\delta R^*}{1+\delta}}{\overline{\theta} q^N - C(q^N) - \frac{\delta R^*}{1+\delta}} \).

As Proposition 3 shows, if the probability of the low-type is sufficiently small, it is better for the seller just to exclude the low-type in the first offer. Because by including it, the seller has to pay the high-type buyer information rent and the gain from the low-type cannot compensate the information rent.

Note that when these two types are getting closer and closer, \( \hat{\pi} \) will become smaller and smaller. In the limit, if \( \overline{\theta} \to \overline{\theta}, \hat{\pi} \to 0 \). This result is quite intuitive. On the other hand, when the difference between these two types is getting larger and larger, \( \hat{\pi} \) will also become larger. However, when \( \frac{\overline{\theta}}{\overline{\theta}} \geq c^0 \), we go back to the efficient outcome case in section 3.2.

According to Proposition 3, as long as \( \pi \) is sufficiently large, the SIE program for the contracts bargaining game looks like that of the contracts design game. Additionally, the bargaining game shrinks to a one-shot game because it ends in the initial time period without any delay. Therefore, we can define \((\overline{m}^*_S, \overline{m}^*_S)\) as the "bargaining-proof contracts" when \( \pi > \hat{\pi} \).

In short, the above analysis tells us that solving the contracts bargaining game is not as complicated as one might think. It is the combination of a screening problem and a signaling problem. First, we solve the signaling problem and find the "Riley outcome", then use the payoffs of "Riley outcome" as the endogenous type-dependent...
continuation payoffs for the PCs in the screening problem. Then we solve the screening problem to get the "bargaining-proof contracts" as long as $\pi$ is sufficiently large. If $\pi$ is sufficiently small, then we follow the same procedure with some slight modification and obtain the SEE.

4.3 Existence of the equilibrium

**Proposition 4** If $0 < \pi \leq \hat{\pi}$, then the SEE exists.

**Proof.** Suppose that $0 < \pi \leq \hat{\pi}$, the following is an equilibrium.

- $S$’s beliefs: Suppose in response to the last offer of $m$ by $S$, $\theta$ offers $\tilde{m}$ after a delay of $\tau$. (1) Suppose $\bar{U}(m) \geq \bar{U}(m^*_B)$. If $\bar{U}(m) \geq \bar{U}(\tilde{m}, \tau)$, $\bar{U}(\tilde{m}, \tau) > \bar{U}(m)$, and $V(\tilde{m}, \tau) \geq V(m^*_B, \tau)$, then $\pi = 1$. Otherwise $\pi = 0$. (2) Suppose $\bar{U}(m) < \bar{U}(m^*_B)$. If $\bar{U}(m^*_B, \tau) \geq \bar{U}(\tilde{m}, \tau)$, $\bar{U}(\tilde{m}, \tau) > \bar{U}(m)$, and $V(\tilde{m}, \tau) \geq V(m^*_B, \tau)$, then $\pi = 1$. Otherwise $\pi = 0$.

- $S$’s strategy: The seller offers $m^*_S$ at time zero. If in response to an offer $m$ by $S$, $\theta$ makes a counteroffer $\tilde{m}$ after delay $\tau$, and $S$’s belief at this node is that $\theta = \bar{\theta}$ with probability $\pi$, then:

  (1) If $\pi = 0$, then the seller accepts $\tilde{m}$ if $V(\tilde{m}, \tau) \geq V(m^*_B, \tau)$, otherwise she rejects it without delay and counteroffers $m^*_S$ right after.

  (2) If $\pi = 1$, then the seller accepts $\tilde{m}$ if $V(\tilde{m}, \tau) \geq V(m^*_B, \tau)$, otherwise she rejects it without delay and counteroffers $m^*_S$ right after.

- $\bar{\theta}$’s strategy: At any point, if $S$ makes an offer $m$, the high-type buyer accepts it without delay if $\bar{U}(m) \geq \bar{U}(m^*_S)$; otherwise he rejects it without delay and counteroffers $m^*_B$ right after.

- $\theta$’s strategy: At any point, if $S$ offers $m$, the low-type buyer accepts it without delay if $\bar{U}(m) \geq \delta \bar{U}(M^*_B)$; otherwise he rejects it without delay and counteroffers $m^*_B$ with delay $\tau^*$. 

18
To verify this equilibrium first note that $S$ would not offer $m$ such that $U(m) < U(m_S^*)$, since given $\theta$’s strategy this leads to a lower payoff than offering $m_S^*$. Since $\pi \leq \hat{\pi}$, by Proposition 3, offering $m_S^*$ is optimal for $S$. The rest of the verification is straightforward. Note that the beliefs above satisfies (A1).

Proposition 5 If $\hat{\pi} < \pi$, then the SIE exists.

Proof. Suppose $\hat{\pi} < \pi$. Then the following is an equilibrium.

- $S$’s beliefs: Suppose that in response to the last offer of $m$ by $S$, $\theta$ offers $\tilde{m}$ after a delay of $\tau$. (1) Suppose $U(m) \geq U(m_S^*)$. If $U(m) \geq U(\tilde{m}, \tau), U(\tilde{m}, \tau) > U(m)$, and $V(\tilde{m}, \tau) \geq V(m_B^*, \tau)$, then $\pi = 1$. Otherwise $\pi = 0$. (2) Suppose $U(m) < U(m_S^*)$. If $U(m_B^*, \tau) \geq U(\tilde{m}, \tau), U(\tilde{m}, \tau) > U(m)$, and $V(\tilde{m}, \tau) \geq V(m_B^*, \tau)$, then $\pi = 1$. Otherwise $\pi = 0$.

- $S$’s strategy: The seller offers $(m_S^*, m_S^*)$ at time zero. If in response to the offer by $S$, $\theta$ makes a counteroffer $\tilde{m}$ after delay $\tau$, and $S$’s belief at this node is that $\theta = \tilde{\theta}$ with probability $\pi$, then:
  (1) If $\pi = 0$, then the seller accepts $\tilde{m}$ if $V(\tilde{m}, \tau) \geq V(m_B^*, \tau)$, otherwise she rejects it without delay and counteroffers $m_B^*$ right after.
  (2) If $\pi = 1$, then the seller accepts $\tilde{m}$ if $V(\tilde{m}, \tau) \geq V(m_B^*, \tau)$, otherwise she rejects it without delay and counteroffers $m_B^*$ right after.

- $\tilde{\theta}$’s strategy: At any point, if $S$ makes a menu offer including a contract $m$, the high-type buyer accepts it and chooses $m$ without delay if $U(m) \geq U(m_B^*)$; otherwise, he rejects it without delay and counteroffers $m_B^*$ right after.

- $\hat{\theta}$’s strategy: At any point, if $S$ offers a menu including a contract $m$, the low-type buyer accepts it and chooses $m$ without delay if $U(m) \geq U(m_B^*)$; otherwise, he rejects it and counteroffers $m_B^*$ with delay $\tau^*$.  

19
To verify this equilibrium, note first that $S$ would not offer $(\overline{m}, m)$ such that $U(\overline{m}) < U(\overline{m}_S^*)$ and $U(m) \geq U(m_S^*)$, since given $\theta$'s strategy this leads to a lower payoff than offering $(\overline{m}_S^*, m_S^*)$. Since $\pi < \pi$, by Proposition 3, offering $(\overline{m}_S^*, m_S^*)$ is optimal for $S$. The rest of the verification is straightforward. Note that the beliefs above satisfies (A1).

\section{Uniqueness of the Equilibrium}

Now we prove the uniqueness of the equilibrium. We begin by deriving a series of lemmas with respect to the boundaries of the players. In addition, the proof is not as complicated as one might think since the "intuitive criterion" has already selected the unique "Riley outcome" for us.

\textbf{Lemma 1} In the bargaining game, $\sup(V) \leq \frac{1}{1+\delta} \overline{R}^*$ and $\inf(U) \geq \frac{\delta}{1+\delta} \overline{R}^*$.

Intuitively, the best situation for the seller is to face a high-type buyer with probability one. In this case, she just offers one contract only to the high-type, and the high-type accepts it without delay. In this situation, she gets the highest payoff $\frac{1}{1+\delta} \overline{R}^*$. For the high-type, the worst case is that the seller knows his type so that he cannot mimic the low-type and he cannot get the information rent. In this situation, he gets his continuation payoff under complete information when the seller moves first, which is $\frac{\delta}{1+\delta} \overline{R}^*$. The following is a proof.

\textbf{Proof.} We prove this Lemma by contradiction. Suppose $V^H = \sup(V) > \frac{1}{1+\delta} \overline{R}^*$. We show $\overline{\theta}$ can profitably deviate by offering $\overline{m}^0(\overline{q}, \overline{p})$ to let $V(\overline{m}^0) = \delta(V^H) + \varepsilon$ for some $\varepsilon > 0$. $S$ will accept this offer. And $\overline{\theta}$'s payoff will be $\delta[\overline{R}^* - \delta(V^H)] - \varepsilon$. Observe that since $V^H > \frac{1}{1+\delta} \overline{R}^*$, we can get $\delta[\overline{R}^* - \delta(V^H)] > [\overline{R}^* - V^H]$. Then there exists an $\varepsilon > 0$ such that the above deviation is profitable for $\overline{\theta}$. This proves $\sup(V) \leq \frac{1}{1+\delta} \overline{R}^*$. Given this, it is easy for us to get $\inf(U) \geq \frac{\delta}{1+\delta} \overline{R}^*$. Simply by telling the truth, $\overline{\theta}$ can guarantee himself at least $\frac{\delta}{1+\delta} \overline{R}^*$.  \hfill  \blacksquare
Lemma 2 $\bar{\theta}$ can get no more than $\frac{1}{1+\delta}R^*$ when he makes an offer.

The high-type has incentive to mimic the low-type. However, any contract other than the separating contract will be rejected by the seller. So the highest payoff the high-type can attain is his complete information payoff when he makes an offer. The following is a proof.

Proof. We show $\sup(\overline{U}) \leq \frac{1}{1+\delta}R^*$ when $\bar{\theta}$ makes an offer. Suppose $\sup(\overline{U}) = \frac{1}{1+\delta}R^* + \alpha$, for some $\alpha > 0$. This implies that $S$ can realize in the respective continuation game a payoff not bigger than $\frac{\delta}{1+\delta}R^* - \alpha$. We show that $S$ can profitably deviate by rejecting and offering a contract $\overline{m}(\overline{q}, \overline{p})$ such that

$$\overline{\theta}q^* - \overline{p} = \delta(\frac{1}{1+\delta}R^* + \alpha) + \frac{1 - \delta^2}{2\delta}$$

With this contract, $\overline{U}(\overline{m}) \geq \delta(\frac{1}{1+\delta}R^* + \alpha)$, $\bar{\theta}$ won’t reject. It remains to be shown that this strategy is strictly profitable to $S$. To see this, note that the difference

$$\delta[\overline{p} - C'(\overline{q}^*)] - \left[ \frac{\delta}{1+\delta}R^* - \alpha \right]$$

$$= \frac{\alpha(1 - \delta^2)}{2} > 0$$

Lemma 3 In the bargaining game, $\sup(\overline{U}) \leq \frac{1}{1+\delta}R^*$ and $\inf(V) \geq \frac{\delta}{1+\delta}R^*$

Intuitively, the low-type has to separate himself from the high-type, so he cannot attain the complete information payoff. The worst case for the seller is that $\pi = 1$.

Proof. The proof is similar to Lemma 1 and is in the appendix.

Lemma 4 If $S$ believes $\pi = 1$, then $S$ accepts any offer no less than $\frac{\delta}{1+\delta}R^*$ for her.

Proof. We want to show $\sup(V) = \frac{\delta}{1+\delta}R^*$ when $\pi = 1$ and when it is $S$’s turn to reject or accept $\bar{\theta}$’s offer. The proof is similar to Lemma 1.

Combining Lemmas 3 and 4, when $\pi = 1$ and when it is $S$’s turn to reject or accept $\bar{\theta}$’s offer, $\inf(V) = \sup(V) = \frac{\delta}{1+\delta}R^*$.
Lemma 5 If $\theta$ signals his type, the unique payoff for him is $U(M_B^*)$ after implementing the "intuitive criterion".

The reason is that the "intuitive criterion" selects the unique "Riley outcome".

Lemma 6 When $0 < \bar{\pi} \leq \bar{\pi}$, S's payoff is bounded from below by $V^{SEE}$.

Proof. For any $\varepsilon > 0$, at time zero $S$ can offer a single contract $m_0(q^*, C(q^*) + \frac{1}{1+\delta} R^* - \varepsilon)$. By Lemma 2, $\overline{\theta}$ accepts the contract. Combining Lemmas 3 and 5 and when $\varepsilon$ is arbitrarily small, $V^{SEE}$ is a lower bound for $S$'s payoff. ■

Lemma 7 When $0 < \bar{\pi} \leq \bar{\pi}$, S's payoff is bounded from above by $V^{SEE}$.

Proof. When $0 < \bar{\pi} \leq \bar{\pi}$, S provides a single contract. Under SEE, she proposes $\overline{\pi}^*$ at time zero, and $\overline{\theta}$ accepts it because of Lemma 2. This contract gives the seller $\frac{1}{1+\delta} R^*$. When $\theta$ counteroffers $U(M_B^*)$, this offer gives the seller $\frac{\delta}{1+\delta} R^*$. From Lemma 1 and 4, we know $V^{SEE}$ is the highest payoff for $S$. ■

Lemma 8 When $\bar{\pi} > \bar{\pi}$, S's payoff is bounded from below by $V^{SIE}$.

Proof. For any $\varepsilon > 0$, $S$ can end the game in time zero by offering a menu $\{m_1(q^N, \theta q^N - \delta U(M_B^*) - \varepsilon), m_1(q^*, \overline{\theta} q^* - (\overline{\theta} - \theta) q^N - \delta U(M_B^*) - \varepsilon)\}$. To see this, note that $\theta$ accepts it by Lemma 5, and $\overline{\theta}$ accepts it by Lemma 2. Since $\varepsilon > 0$ can be chosen to be arbitrarily small, $V^{SIE}$ indeed constitutes a lower bound for $S$'s payoff. ■

Lemma 9 When $\bar{\pi} > \bar{\pi}$, S's payoff is bounded from above by $V^{SIE}$.

Proof. When $\bar{\pi} > \bar{\pi}$, S proposes a menu. In the SIE program, $S$ makes a take-it-or-leave-it menu offer and delay is costly, thus $V^{SIE}$ is the highest possible payoff for $S$. ■

Proposition 6 If $0 < \bar{\pi} \leq \bar{\pi}$, then the only equilibrium path is the SEE.
Proof. Combining Lemmas 6 and 7, $V^{SEE}$ is the unique equilibrium payoff for $S$ when $0 < \pi \leq \hat{\pi}$. Also $\theta$ achieves his maximum payoff when signaling his type according to Lemma 5 and $\bar{\theta}$ attains his maximum payoff when he makes an offer (Lemma 2). Thus, the SEE is the only candidate for an equilibrium path. The existence of an equilibrium that supports this path is established in Proposition 4. ■

Proposition 7 If $\hat{\pi} < \pi$, the only equilibrium path is the SIE.

Proof. Combining Lemmas 8 and 9, $V^{SIE}$ is the unique equilibrium payoff for $S$ when $\hat{\pi} < \pi$. Also under the equilibrium path, $\bar{\theta}$ has a higher payoff than when he proposes and $\theta$ gets the unique payoff as Lemma 5 shows. Thus, the SIE is the only candidate for an equilibrium path. The existence of an equilibrium that supports this path is established in Proposition 5. ■

5.1 Multi-dimensionality of bargaining and the uniqueness of the equilibrium

Much of the standard bargaining literature has a large multiplicity of equilibria even after applying all kinds of refinements. For example, Admati and Perry (1987), which have the same bargaining rule as this paper and also use the "intuitive criterion", still have multiple equilibria when the parameter values are in the intermediate ranges. However, this research obtains the unique equilibrium with the implementation of the "intuitive criterion", and sometimes even without the refinement (the efficient outcome with incomplete information in section 3.2) with regard to different parameter values.

This difference of results comes from the bargaining dimensionality. As we mentioned before, the standard bargaining literature usually involves only one-dimensional bargaining, while this research is about multi-dimensional bargaining. Contrary to the thought that multi-dimensionality would make the multiple equilibria problem even worse, we actually can get some very neat results and the unique equilibrium. The reason is that if there is correlation between those multiple bargaining dimensions, the
players can combine those dimensions as sorting device, either signaling or screening credibly, and the uninformed player can also provide menus to let the informed party to self-select. With those extra sorting variables, the incomplete information game can quickly become the complete information one. This is impossible in the standard bargaining literature.

There is a small literature about multi-dimensional bargaining and they all yield some very striking results. Wang has (1998) discussed the one-sided offer case in which an uninformed firm and an informed worker bargain over two dimensions, output quality and wage. Only the firm offers a menu of quality-wage combination each period. He has shown that the uninformed party proposes the same take-it-or-leave-it menu every period and the unique sequential equilibrium outcome is separating without delay. He has claimed that "in multi-dimensional bargaining, the Coase Conjecture holds in the sense that the game ends in the first period. But it fails in the sense that the uninformed party can preserve the entire bargaining power" (Wang, 1998, page 295). Inderst (2003) and Sen (2000) investigated exogenous alternating-offer cases. Both of them have shown that for a subset of parameters the bargaining game has a unique equilibrium where efficient contracts are implemented in the first period. We achieve a similar efficient outcome in section 3.2. Inderst (2003) has allowed both sides to propose menus and only focused on the efficient outcome. Sen (2000) has also discussed the inefficient outcomes and combined the refinement of Perfect Sequential Equilibrium (Grossman and Perry, 1986) and "elimination of never weak best response" (Kohlberg and Mertens, 1986) to prune the equilibrium set.

6 More Than Two Types

Now we extend our model to cases with more than two types. First we discuss the case with finite number of types. Then we analyze the case with a continuum of types.
6.1 Finite number of types

We assume the buyer has a finite number of types: $\theta_1 < \theta_2 < \ldots < \theta_n$, with $n \geq 3$ and with probabilities $\sum_{i=1}^n \pi_i = 1$. For simplicity, we assume $\theta_{i+1} - \theta_i = \Delta < c^0$, $i = 1 \ldots n - 1$, and the hazard rate $h(\theta_k) = \frac{\pi_k}{1 - \sum_{i=1}^{\pi_k} \pi_i}$ is nondecreasing.

As the previous analysis, we first focus on the Riley outcome,\(^{15}\) which can be obtained by successively solving the following programs:

Program for $\theta_n$:

\[
\max_{\{q_n, p_n\}} U_n = \theta_n q_n - p_n \tag{7}
\]

s.t. $p_n - C(q_n) \geq \frac{\delta}{1 + \delta} R^*_n \tag{PC_n}$

Program for $\theta_k$, $k = 1 \ldots n - 1$:

\[
\max_{\{q_k, p_k\}} U_k = \delta^* [\theta_k q_k - p_k] \tag{8}
\]

s.t. $p_k - C(q_k) \geq \frac{\delta}{1 + \delta} R^*_k \tag{PC_k}$

\[
\theta_{k+1} q_{k+1} - p_{k+1} \geq \delta^* (\theta_{k+1} q_k - p_k) \tag{IC_{k+1}}
\]

Since the single-crossing property holds in our model, we only need to consider the local downward incentive constraints. As we mentioned before, there can be interior solutions as well as "corner solutions" depending on the parameter values. In particular, if those lowest types who cannot afford to distort their qualities downward will signal their types only by strategic delays.

\(^{15}\)The "intuitive criterion" still selects the Riley outcome here. See Cho and Kreps (1987, page 214) for a discussion: "But when the Intuitive Criterion is applied to this game, one can show that the unique equilibrium outcome that survives is the Riley outcome, no matter how many types there are (as long as the number is finite)." Our contracts bargaining game belongs to the category of "this game".
Solving the problems, we can find each type’s contract $M_i^s$ and his corresponding utility under this contract $U_i(M_i^s), i = 1, ..., n$. We can see that only the highest type $\theta_n$ obtains the efficient outcome, that is, the first-best quality without delay, other types have the inefficient outcomes involving either quality distortion or delay or both.

Now we go back to the seller’s problem. Since the model has the single-crossing property, only the local downward incentive constraints are binding when the monotonicity condition $q_{i-1} \leq q_i$ holds, so that the seller’s problem can be written as:

$$\max_{\{(q_i, p_i)\}} \sum_{i=r}^{n} \pi_i (p_i - C(q_i)) \quad (9)$$

s.t. $\theta_r q_r - p_r = U_r(M_r^s)$ (PC$_r$)

$\theta_{i+1} q_{i+1} - p_{i+1} = \theta_{i+1} q_i - p_i$ for $i = r, ..., n - 1$ (LDIC)

$q_i \geq q_j$ where $\theta_i > \theta_j \geq \theta_r$ (MC)

The assumption that the hazard rate $h$ is nondecreasing is sufficient to imply that $q_{i-1} < q_i$.

Notice that the seller doesn’t necessary include all the types in her menu. To see the reason, let’s first ignore constraint (PC$_r$) and solve the program. We will have:

$$C'(q_n) = \theta_n$$

$$C'(q_k) = \theta_k - \frac{1}{h(\theta_k)}(\theta_{k+1} - \theta_k) \quad \text{for } k = 1, ..., n - 1.$$

For $\theta_k$ to be included in the menu, it is necessary that $C'(q_k) \geq 0$, that is, $\theta_k \geq \frac{\Delta}{h(\theta_k)}$.

For some lowest types, this condition may not hold and those types will be excluded in the seller’s menu for sure.

However, $\theta_k \geq \frac{\Delta}{h(\theta_k)}$ is necessary but not sufficient condition for $\theta_k$ to be included in the seller’s menu. Seller here faces a trade-off. On one hand, to include a specific type, she has to give up the information rent; on the other hand, to exclude that type,
she has to bear the delay cost. She can find the threshold type \( \theta_r \), which makes her indifferent, and she will include all \( \theta_i \geq \theta_r \) in her menu. The following Proposition summarize the above discussion:

**Proposition 8** There exists a \( \theta_r \) such that if \( \theta_i \geq \theta_r \), \( \theta_i \) will be included in the seller’s menu and be separated without delay and the menu is the solution to program (9); on the other hand, if \( \theta_i < \theta_r \), \( \theta_i \) will not be included in the seller’s menu and will separate himself by his least-cost-separating contract involving delay, which is included in the solutions to program (7) and (8).

Note that for all \( \theta_i \geq \theta_r \), their contacts are ordering in terms of qualities, that is, higher type gets higher quality and only the highest type gets the efficient quality. For all \( \theta_i < \theta_r \), their contracts are ordering in terms of delay, that is, the lower is the type, the longer is the delay and the lowest type’s delay is the longest.

### 6.2 A continuum of types

We now turn to the case with a continuum of types. Suppose that the buyer’s type \( \theta \in [\underline{\theta}, \bar{\theta}] \) with p.d.f. \( f(\theta) \) and c.d.f. \( F(\theta) \). We also assume that \( h(\theta) = \frac{f(\theta)}{1 - F(\theta)} \) is nondecreasing.

As before, we first solve the buyer’s signaling problem and focus on the "least-cost-separating contracts" \( M(\theta) \),\(^{16}\) and then take the corresponding utility \( U(M(\theta)) \) as the outside option for each type when we continue to solve the seller’s screening problem. The seller’s problem can be written as

\[
\max_{\{(q(\theta), p(\theta))\}} \int_{\underline{\theta}}^{\bar{\theta}} [p(\theta) - C(q(\theta))] f(\theta) d\theta \tag{10}
\]

subject to \( \hat{\theta} q(\hat{\theta}) - p(\hat{\theta}) = U(M(\hat{\theta})) \) \hspace{1cm} (PC)

\(^{16}\)We are not sure what kind of refinement can help us to select the "Riley outcome" in the continuous-type signaling problem.
\[ p'(\theta) = \theta q'(\theta) \quad \text{for} \quad \theta \in [\hat{\theta}, \overline{\theta}] \]  

(LDIC)

\[ \frac{dq(\theta)}{d\theta} \geq 0 \quad \text{for} \quad \theta \in [\hat{\theta}, \overline{\theta}] \]  

(MC)

Program (10) is similar to program (9). By the same token, we can find a \( \hat{\theta} \) such that when \( \theta \in [\hat{\theta}, \overline{\theta}] \), \( \theta \) will be included in the seller’s menu and be separated without delay and program (10) provides the menu; on the other hand, when \( \theta \in [\hat{\theta}, \overline{\theta}] \), \( \theta \) will not be included in the seller’s menu and will separate himself by his least-cost-separating contract involving delay.

7 Extension and Conclusion

There are some extensions on the model.

Consider the game with two types in which the buyer makes the initial offer at time zero. It can be shown, using the similar arguments to those used in the previous sections, that if \( \frac{\bar{b}}{\bar{b}} \geq c^0 \), buyer \( \hat{\theta} \) (\( \theta \)) proposes \( m_B^\ast \) (\( m_B^\ast \)) at time zero, the seller accepts it, and the game ends at time zero. Given \( \frac{\bar{b}}{\bar{b}} < c^0 \), there is one unique separating equilibrium path where \( \hat{\theta} \) offers \( m_B^\ast \) at time zero while \( \theta \) offers \( m_B^\ast \) with strategic delay \( \tau^* \), and \( S \) accepts either of them without delay.

Meanwhile, following Myerson (1983), Maskin and Tirole (1990), and some bargaining literature (Cramton, 1992), this research can be extended to a "two-sided private information" case. In addition, many other bargaining rules as well as various bargaining power distributions can be implemented rather than the one we have used.

Despite the fact that many real-life contract negotiations involve bargaining, standard contract theory has not included bargaining in its model until now. The standard theory abstracts away bargaining by assuming that the principal has all of the bargaining power and she makes a take-it-or-leave-it offer. This research introduces sequential bargaining into the non-linear pricing model by allowing alternating offers and strategic delay. When the difference between the types is sufficiently large, we have obtained

\[ ^{17} \tau^* \text{ could be } 0. \]
the efficient outcome. When the difference between the two types is not sufficiently large, if the initial probability for the high-type is sufficiently large, we have achieved the sequential separating equilibrium; otherwise, we have had the simultaneous separating equilibrium. We have proved the existence and the uniqueness of the SEE and the SIE. We have defined the "dynamic least-cost separating contracts" as well as the "bargaining-proof contracts". We have claimed that introducing multiple dimensions helps to resolve the multiple equilibria problem of the standard bargaining theory. We have shown that the infinite-horizon contracts bargaining game shrinks to a one-shot contract design game with the exception that the right-hand sides of one-shot contract design game are replaced by the endogenous type-dependent continuation payoffs generated by the bargaining process under certain conditions.

The results of this paper are ready to be applied to various contracts bargaining and design environments. For example, the firm and the labor union bargain over the labor contracts which specify the wage and the workload; the borrower and the lender bargain over the financial contracts which specify the interest rate and the size of the loan; and the insurance company and the customer bargain over the insurance contracts which specify the premium and the benefit.

8 Appendix

Proof of Lemma 3.

Proof. Suppose $U^H = \sup(U) > \frac{1}{1+\delta} R^*$. We show $S$ can profitably deviate by offering $m^0(q, p)$ to let $U(m^0) = \delta(U^H) + \xi$ for some $\xi > 0$. $S$ will accept this offer. And $S$’s payoff will be $\delta[R^* - \delta(U^H) - \xi]$. Observe since $U^H > \frac{1}{1+\delta} R^*$, we can get $\delta[R^* - \delta(U^H)] > [R^* - U^H]$. Then there exists an $\xi > 0$ such that the above deviation is profitable for $S$. This proves $\sup(U) \leq \frac{1}{1+\delta} R^*$. Given this, it is easy to get $\inf(V) \geq \frac{\delta}{1+\delta} R^*$. Note instead of accepting an offer $m^0$ giving him $V(m^0) < \frac{\delta}{1+\delta} R^*$, $S$ can always reject it right away and propose $m^*_S$. Both buyers will accept it with probability one. $S$ can get at least $\frac{\delta}{1+\delta} R^*$ in this worst case. □
References


